THEORETICAL INVESTIGATION OF WALL EFFECT ON DRAG COEFFICIENT OF DIFFERENT PARTICLES SHAPE MOVING IN NON-NEWTONIAN FLUIDS

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ABSTRACT

In this study ,the settling velocity for spherical and non-spherical particle vertically settling in non-Newtonian fluid .The settling velocity calculated by derived the equation for motion particle and this equation solved by MATLAB program ,then this value supported to ANSYS FLUENT 15.0 to evaluate the drag force on the particles .The effect of finite wall on the particle expressed by settling particle along the axis of cylindrical tubes .The governing equation and boundary condition were numerically solved in a wide range of diameter ratio (0.118-0.814),the result show that the finite wall lead to increase in the drag force and the settling velocity depend on sphericity and diameter for particle ,this investigation important to reduce the hydrodynamic drag in many application.

INTRODUCTION

The problem of describing the settling velocity of a falling spherical and non-spherical particle in non-Newtonian fluid is important in many application ,In unit operation such as centrifugal and gravity collection and separation also viscosity measurements in non-Newtonian fluid using the falling ball method.It is also necessary to know the time distance required to reach the terminal velocity, Clift et al(1978) .In this state of non-spherical particles less information founded in the literature survey when the particle settling in the fluid subjected to different force gravitational force (FG),drag force (FD),and buoyancy force (FB) ,this force when reach to equilibrium happen the terminal velocity,Brown Phillip.P(2003).When the particle start moving in the fluid ,a velocity dependent on drag force start subjected on it.This velocity dependent on drag coefficient of the particle ,it is defined by;

$$CD = \frac{F_D}{\frac{1}{2}\,\rho U^2 \frac{\pi}{4} Dn^2}$$

Where FD;is the drag force ,ρ: density of fluid , Dn:equivalent diameter ,U:velocity of the particle.For spherical particles the drag force depend on the diameter of particle and density as well as the viscosity of the fluid , for non-spherical particles the drag force depend on the shape of the object and its orientation when it moves in the fluid.To evaluate the velocity for falling particle in the fluid ,the relationship between Reynolds number and drag coefficient is required ,use correlation by Chien (1994), this correlation used in ranges of $(0.2 \le Ø \le 1)$ and Reynolds number $(Re<50000)$ for different shape particles.

$$CD = \frac{30}{Re} + 67.289e^{-5.03\emptyset}$$

A sphere remains set inside cylinder which settling along the axis with a uniform velocity ,this velocity is taken as the terminal of sphere in the liquid this way for solution used by previous studies (Beris et al ,1985;Blackery&Mitsoulis1997 ;Yu&Wachs2007). The wall correction factor effect on the particle motion can be expressed in different ways:as the drag force in finite fluid to infinite fluid ratio ,velocity ratio for bounded to unbounded medium ,viscosity ratio and drag coefficient ratio ,thus the wall effect typically represented in term of wall correction factor $^{v_t}/_{v_{t\infty}}$,$^{FD}/_{FD_\infty}$,$^{CD}/_{CD_\infty}$.

THEORETICAL MODEL

Figure (1) show the forces subjected to the particle when falling in fluid. Equation of motion for particle in fluid that can be used for non- Newtonian fluid is derived in this study depend on taken free body diagram for the particle that falling in fluid and specified the initial condition. Assume a particle with mass(*m)* settling from an initial height and let it settling freely the forces effected on it after its steady are force gravity , drag force and buoyancy through which it settling:

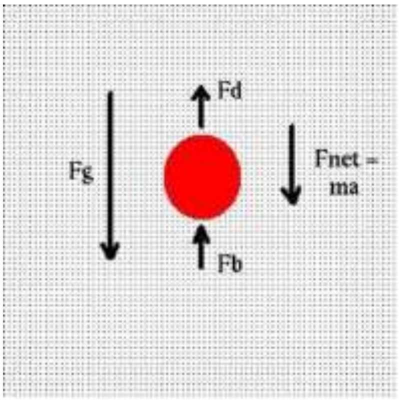


Figure (1) free body diagram for particle settling in fluid

Assumption used to driving the equation of motion for particles settling in non-Newtonian fluid:

1) Particle does not change orientation
2) Fluid static.
3) Particle moving on gravity effect .
4) Rigid spherical and non- spherical particle settling in an infinite of an incompressible fluid.
5) Particle falling in fluids light assuming ($\rho \ll \rho_p$) basset history force neglect (C_A=0.5)

Governing equation to driving equation of motion for particles:

The force exerted by gravity is;

$$F_G = m * g \tag{1}$$

The force exerted by fluid resistance;

$$CD = \frac{F_D}{\frac{1}{2}\rho U^2 \frac{\pi}{4} Dn^2} \tag{2}$$

The analytical correlation between drag coefficient and Reynolds number for spherical and non-spherical particle presented by S.F.chien (1994);

$$CD = \frac{30}{Re} + 67.289 e^{-5.03\emptyset} \tag{3}$$

This equation was stated for range of 0.2≤Ø≤1 and Re≤ 5*10^4

Driving equation for non-Newtonian fluid:

Equating (2) and (3);

For non-Newtonian fluid:

$$F_D = 11.77 K Dn^{2-n} U^n + 26.41 \rho U^2 Dn^2 e^{(-5.03\emptyset)} \tag{4}$$

$$F_{BE} = \frac{1}{6}\pi Dn^3 \rho C_A \frac{du}{dt} \tag{5}$$

$\rho \ll \rho_s$ Basset history force neglect $C_A = 0.5$

The force exerted by buoyancy is:

$$F_B = \rho\, v\, g$$

Where ;

$$v = \frac{m}{\rho_p}$$

Then ; Newton's Second Law of Motion says 'that the total force acting on an object equals the mass of the object times its acceleration *a"*.

Thus;

$$F = F_G - F_B - F_D\text{-}F_{BE} \qquad (6)$$

For Non-Newtonian fluid:

$$m\frac{dv}{dt} = mg - \frac{m\rho g}{\rho_p} - 11.77k\ Dn^{1.264}\ U^{0.736} - 26.41\rho U^2 Dn^2 e^{-5.03\emptyset} - \frac{1}{6}\pi Dn^3 \rho C_A \frac{dv}{dt} \qquad (7)$$

Since the object is not moving at (time =0).it is possible to solve this function v above.

Carboxy methyl cellulose (CMC) was used as non-Newtonian fluid and having:

n=0.736 k= 0.182 (pa.s^n) μ= 15 (c.p) ρ = 1042 (Kg/m^3)

Use MATLAB program to evaluate velocity

To solve the equation of motion that used to evaluate terminal velocity for each particle theoretically can be used by built-in Matlab program to solve function (ode45). The expression of this function is:

```
[t, v] = ode45(@aux_function, time, initial_condition);
```

Then use the editor Matlab to create a file (vprime.m) as shown below describe for this program:

```
function vp = vprime(t,v)
%  function vp = vprime(t,v)
% compute dv/dt
m=0.0055;%mass for particle;
g=9.81; % acceleration constant;
a=1040; %density for fluid;
b=7800;%density for particle;
c=1; % sphericity for particle;
d=0.011;%  nominal diameter for particle ;
e=0.182;%consistency index for fluid;
f=0.5;%integrated added mass coefficient ;
vp=[m*g*(1-(a/b))-26.41.*a*d^2*(exp(-5.03*c))*v.^2-11.77*e*d^1.264*v.^0.736]./(m+0.523*d^3*a*f);
%equation for  non-Newtonian fluid;
>>time=[0 10];%time interval put in commend window;
>> initialconditions=0;%value for initial condition for velocity and time ;
>> fname='vprime';
>> [t,v]=ode45(fname,time,initialconditions)
```

GEOMETRY OF SPHERICAL AND NON-SPHERICAL PARTICLES

Wadell, H. (1933) was the first researcher described the sphericity.While the sphericity is the ratio of surface Area of sphere having a volume equal to the volume of the particle to Area of particle.

$$\Psi = \frac{A_s}{A_p} \qquad (8)$$

The dimension of non-spherical particle to calculate sphericity:

- The nominal diameter (Dn) is the diameter of sphere have a volume equal to the volume of particle.

$$V_{sphere} = \frac{1}{6}\pi\, Dn^3 \qquad (9)$$

$$\frac{1}{6}\pi\, D_n^3 = Vparticle \qquad (10)$$

$$D_n = \sqrt[3]{\frac{6\,V_p}{\pi}} \qquad (11)$$

- Surface Area (A_s) equale surface area of sphere having a volume equal to the volume of the particle .

It was calculated the equivalent diameter (Dn)

$$A_s = \pi\, Dn^2 \qquad (12)$$

Sample of calculation

sphere

Table (1): parameters for sphere particles

Shape No.	Diameter mm	Weight g	Density kg/m^3
1	11	5.5	7800
2	28.5	96	

Cylinder

$A_p = 2 * \frac{\pi}{4} D^2 + \pi D l$

$V_p = \frac{\pi}{4} D^2 l$

D: diameter of the cylinder base
l: length of cylinder

Table (2): parameters for cylinder particles

l(m)$*10^{-2}$	D(m)$*10^{-2}$	A_p (m^2)	V_P (m^3)	Dn (m)	A_s(m^2)	Ψ
0.89	1	$4.364*10^{-4}$	$7.05*10^{-7}$	$11*10^{-3}$	$3.79*10^{-4}$	0.87
3.89	2	$3.07* 10^{-3}$	1.22410^{-5}	$28.5*10^{-3}$	2.5510^{-3}	0.830

cube

$A_p = 6 * l_c^2$

$V_p = l^3$

l_c: length of cubic side

Table (3): parameters for cube particles

l (m)$*10^{-2}$	A_p (m^2)	V_P (m^3)	D_n (m)	A_s(m^2)	Ψ
0.89	$4.752* 10^{-4}$	$7.05*10^{-7}$	$11*10^{-3}$	$3.799* 10^{-4}$	0.799
2.3	$3.174* 10^{-3}$	1.22410^{-5}	$28.5*10^{-3}$	$2.55* 10^{-3}$	0.803

Cone

$A_p = \frac{\pi}{2} D h_s + \frac{\pi}{4} d D^2$

$V_p = \frac{\pi}{12} D^2 h$

D: diameter of the Cone base , h: high of Cone, h_s: the length of slant

Table (4): parameters for cone particles

D (cm)	h(cm)	h_s(m)	A_p (m^2)	V_P (m^3)	D_n (m)	A_s (m^2)	Ψ
1	2.7	0.0274	$5.085* 10^{-4}$	$7.05* 10^{-7}$	$11*10^{-3}$	$3.799* 10^{-4}$	0.747
3.42	4	0.0435	$3.25* 10^{-3}$	$1.224\ 10^{-5}$	$28.5* 10^{-3}$	$2.55* 10^{-3}$	0.783

Rectangular

$A_P = 2(W * L) + 2(L * H) + 2(H * W)$

$V_P = W * L * H$

W:width of rectangular
L:length of rectangular
H:hight of rectangular

Table (5):parameters for Rectangular particles

W$*$ 10^{-2} m	L$*$ 10^{-2} m	H$*$ 10^{-2} m	A_P (m^2)	V_P (m^3)	D_n (m)	A_S (m^2)	Ψ
2	0.7	0.5	5.510^{-4}	$7.05*10^{-7}$	$11*10^{-3}$	3.79910^{-4}	0.69
2	3.06	2	3.2410^{-3}	1.22410^{-5}	$28.5*10^{-3}$	2.5510^{-3}	0.785

USE ANSYS FLUENT 15.0 FOR SIMULATION MATHEMATICAL MODELING

Commercial package of FLUENT 15.0 under ANSYS 15.0 software, after graphing the geometry by AUTOCAD 2014 and describe the mesh model by using ANSYS meshing 15.0. The system geometry used in the CFD model has the dimensions:

- Fluid zone cylinder (93,75,55,35) mm inside diameter,(100,80,60,40)mm outer diameter and 1000 mm of high .
- Particle zone: different conflagration in different dimension used to calculate mathematical result for this simulation .Each particle draw in the four tubes and then import to CFD to evaluate simulation.

Consider a particle with nominal diameter (Dn)located at the axis of a cylindrical tube having a diameter (D)falls at steady velocity (vo)take from the privous section (Vt)terminal velocity, Prashant, J. J. Derksen(2010), for particle with incompressible fluid this situation is equal to the fluid moving with cylinder wall at a uniform velocity (V0=Vt) as shown in figure (2).

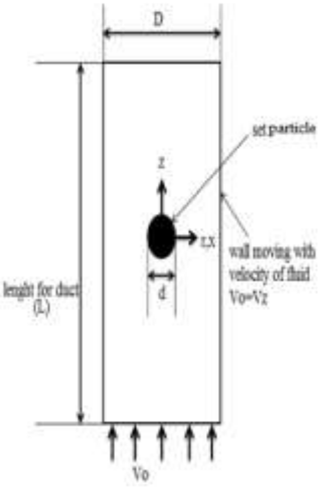


Figure (2) falling particle in tube with boundary conditions

Conservation equations of momentum and continuity for turbulent model of the flow are presented in **FLUENT** built – in solver.

The Mass Conservation (Continuity) Equation:

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho\,\vec{v}) = 0 \qquad (13)$$

$Where;$

$\vec{v}$: is the velocity vector:

$$\nabla.\vec{v} = 0 \qquad (14)$$

Momentum Conservation Equations:

$$\frac{\partial\rho}{\partial t} + (\rho\vec{v}) + \nabla.(\rho\vec{v}\vec{v}) = -\nabla p + \nabla.(\bar{\bar{\tau}}) + \rho\vec{g} + \vec{F} \qquad (15)$$

$$\bar{\bar{\tau}} = \mu\,[(\nabla\vec{v} + \nabla\vec{v}^T - \tfrac{2}{3}\nabla.\vec{v}I] \qquad (16)$$

For an axisymmetric, steady flow the boundary conditions are:

1) Inlet Vr=0 ,Vz=V0
2) Outlet p=0 ,$\frac{dv}{dz} = 0$
3) Symmetry Vr=0
4) Sphere wall Vr=Vz=0
5) Tube wall Vr=0,Vz=V0

At this above governing equation along with the boundary condition are numerically solved this is useful to introduce some parameters such as drag force also the velocity value is adjusted in such away for all measure of tubes and study the effect of the wall on the drag force.

Wall correction factor in this study calculation from the ratio for drag force in bounded media to drag force in unbounded media:

$$FW = \frac{FD}{FD_{\infty}}$$

RESULT AND DISCUSSION

Behavior of settling velocity

From figures (3) and (4) show that the relationship between velocities with equivalent diameter is proportional for the same shape. Also it shows the velocity of sphere more than others particles because of the sphericity. So the large particles diameter give the highest terminal velocity than smaller particles for the same shape, the reason could be the large particles have the highest weight with the less affected by viscous force and buoyancy force opposing the settling velocity. But for different shapes the sphericity effects of velocity more than the weight. The velocities of falling particles in non-Newtonian fluids were increased.

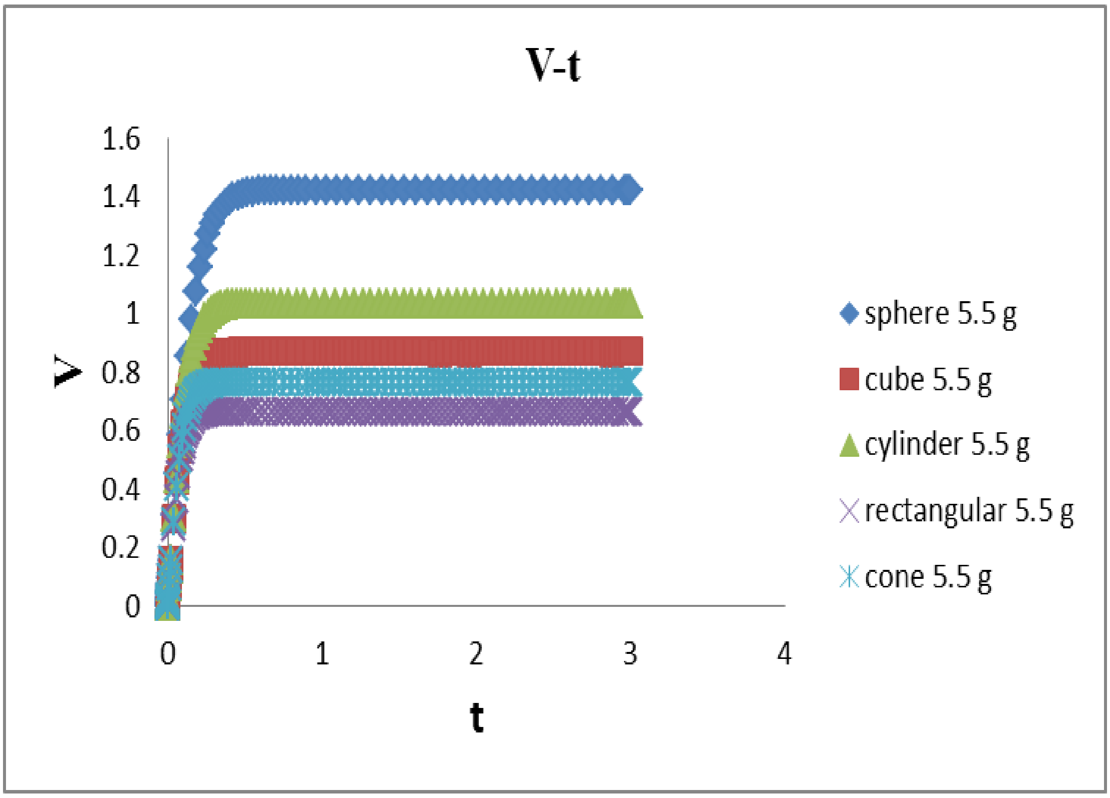


Figure (3): relationship between velocity with time for 5.5 g weight particles

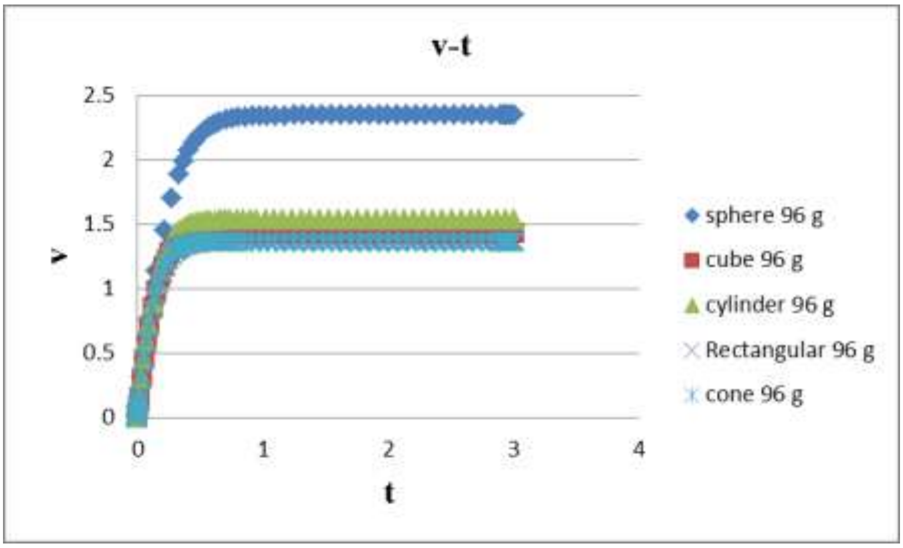


Figure (4): relationship between velocity with time for 96 g weight particles

Behavior of wall effect:

From figures (5) up to (9) show the wall correction factor increase with increasing diameter ratio the wall correction factor calculated from the ratio for drag force in bounded media shown in table (6)&(7) to the drag force in unbounded media evaluated from extrapolating to data for drag force in bounded media to(d/D=0) , the drag force increase with increasing diameter ratio because the gap between wall container and particle surface decrease this lead to relative velocity around particle decrease so the drag force increase.

Table (6): numerical results from ANSYS FLUENT for sphere and cylinder particles

sphere				cylinder			
Sample1		Sample2		Sample1		Sample2	
FD	d/D	FD	d/D	FD	d/D	FD	d/D
0.028	0.118	0.496	0.306	0.039	0.118	0.36	0.306
0.029	0.146	0.634	0.38	0.04	0.146	0.39	0.38
0.03	0.2	0.849	0.518	0.041	0.2	0.48	0.518
0.034	0.314	4.762	0.814	0.047	0.314	1.01	0.814
0.0258	0	0.110	0	0.037	0	0.31	0

Table (7): numerical results from ANSYS FLUENT for cube, cone and rectangular particles

cube				cone				rectangular			
Sample1		Sample2		Sample1		Sample2		Sample1		Sample2	
FD	d/D	FD	d/D	FD	d/D	FD	d/D	FD	d/D	FD	d/D
0.028	0.118	0.66	0.306	0.0074	0.118	0.40	0.306	0.0080	0.118	0.45	0.306
0.028	0.146	0.79	0.38	0.0074	0.146	0.46	0.38	0.0087	0.146	0.49	0.38
0.034	0.2	1.07	0.518	0.0077	0.2	0.76	0.518	0.0080	0.2	0.62	0.518
0.030	0.314	4.46	0.814	0.0097	0.314	8.03	0.814	0.0086	0.314	1.61	0.814
0.028	0	0.52	0	0.007	0	0.4	0	0.008	0	0.399	0

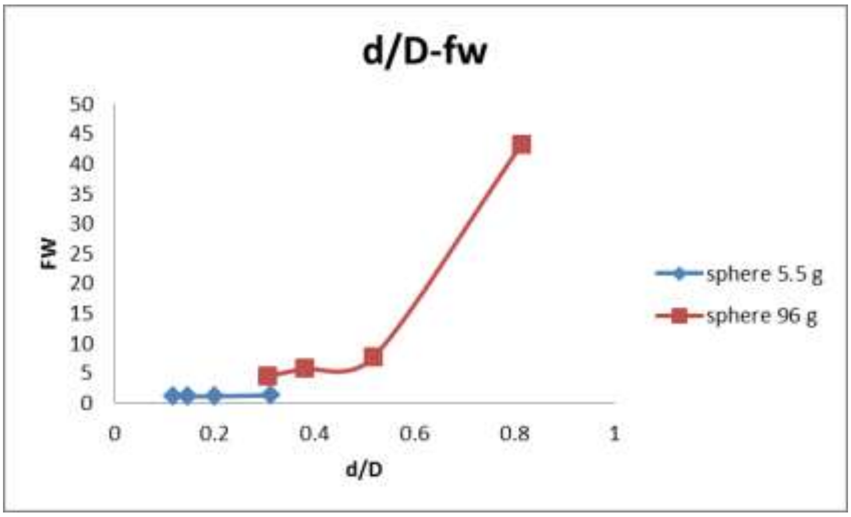


Figure (5): Relationship of (FW-d/D) for falling sphere in CMC

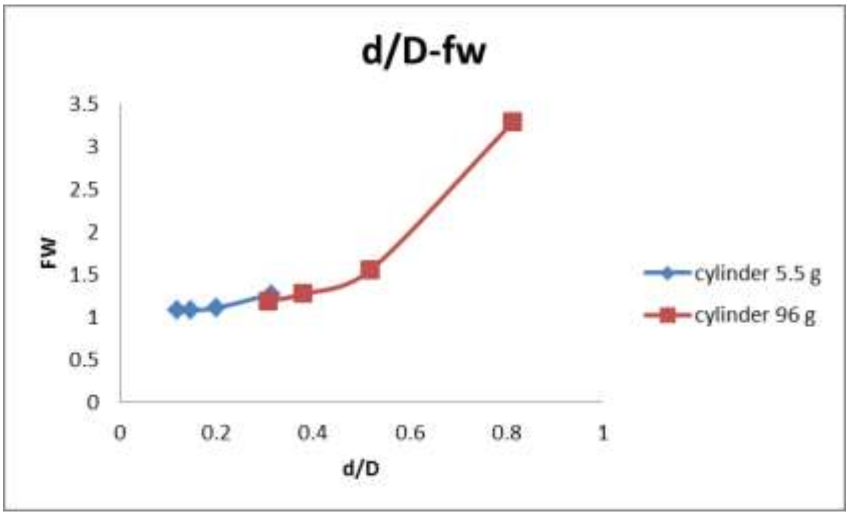


Figure (6): Relationship of (FW-d/D) for falling cylinder in CMC

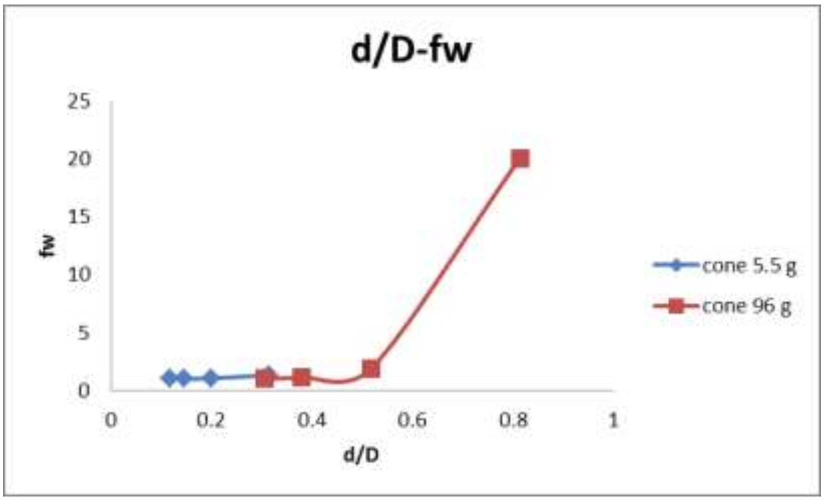


Figure (7): Relationship of (FW-d/D) for falling cone in CMC

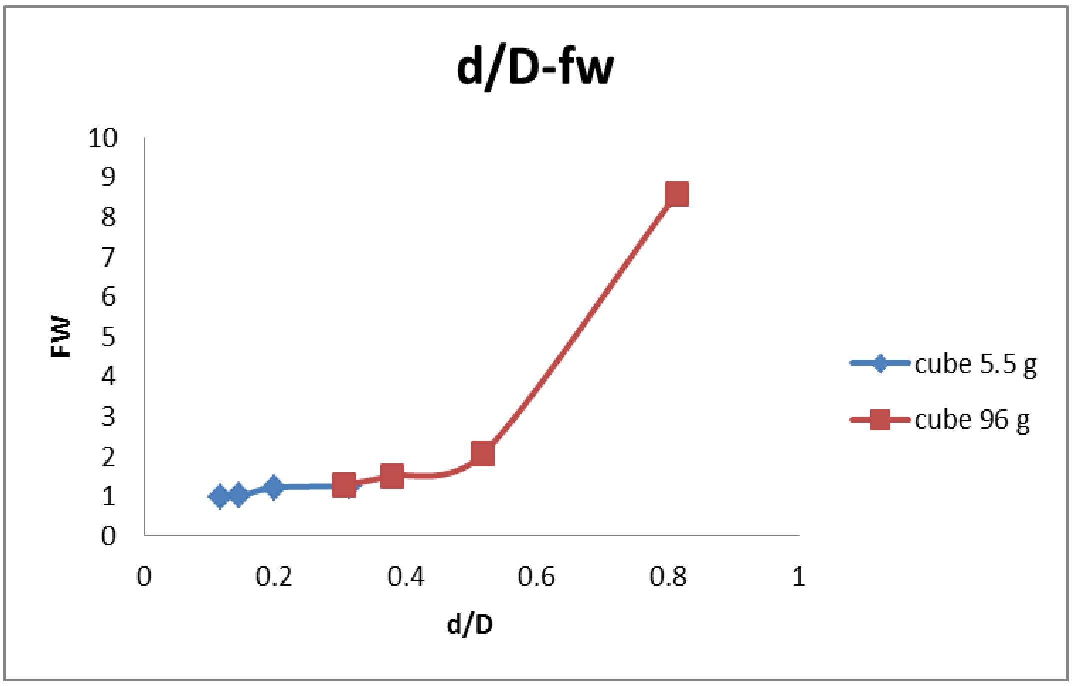


Figure (8): Relationship of (FW-d/D) for falling cube in CMC

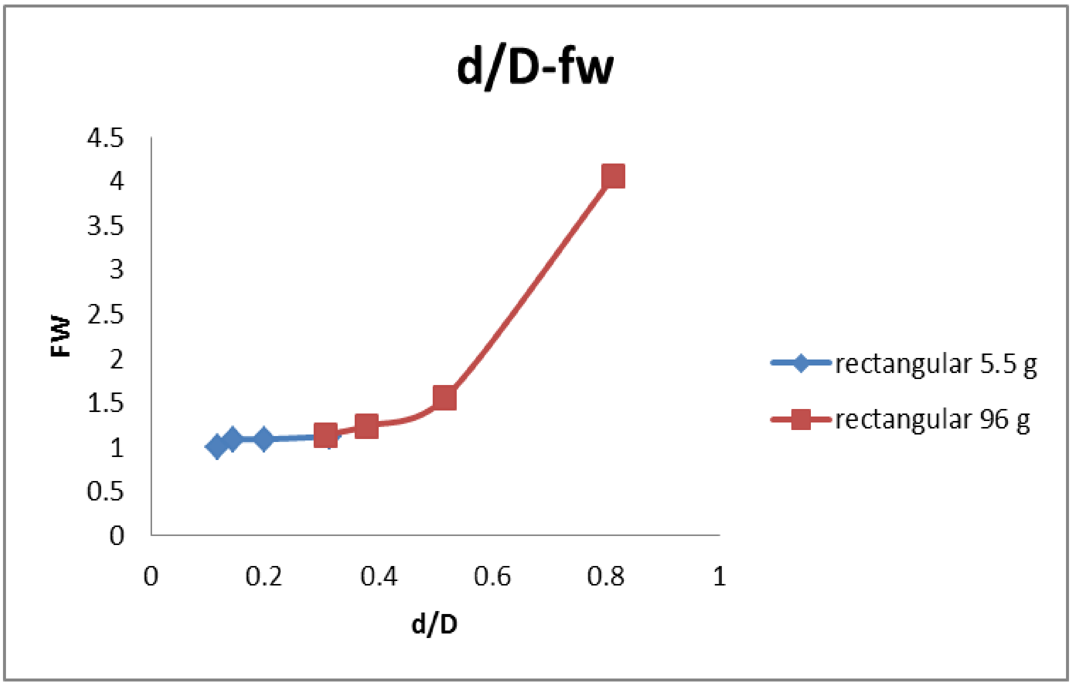
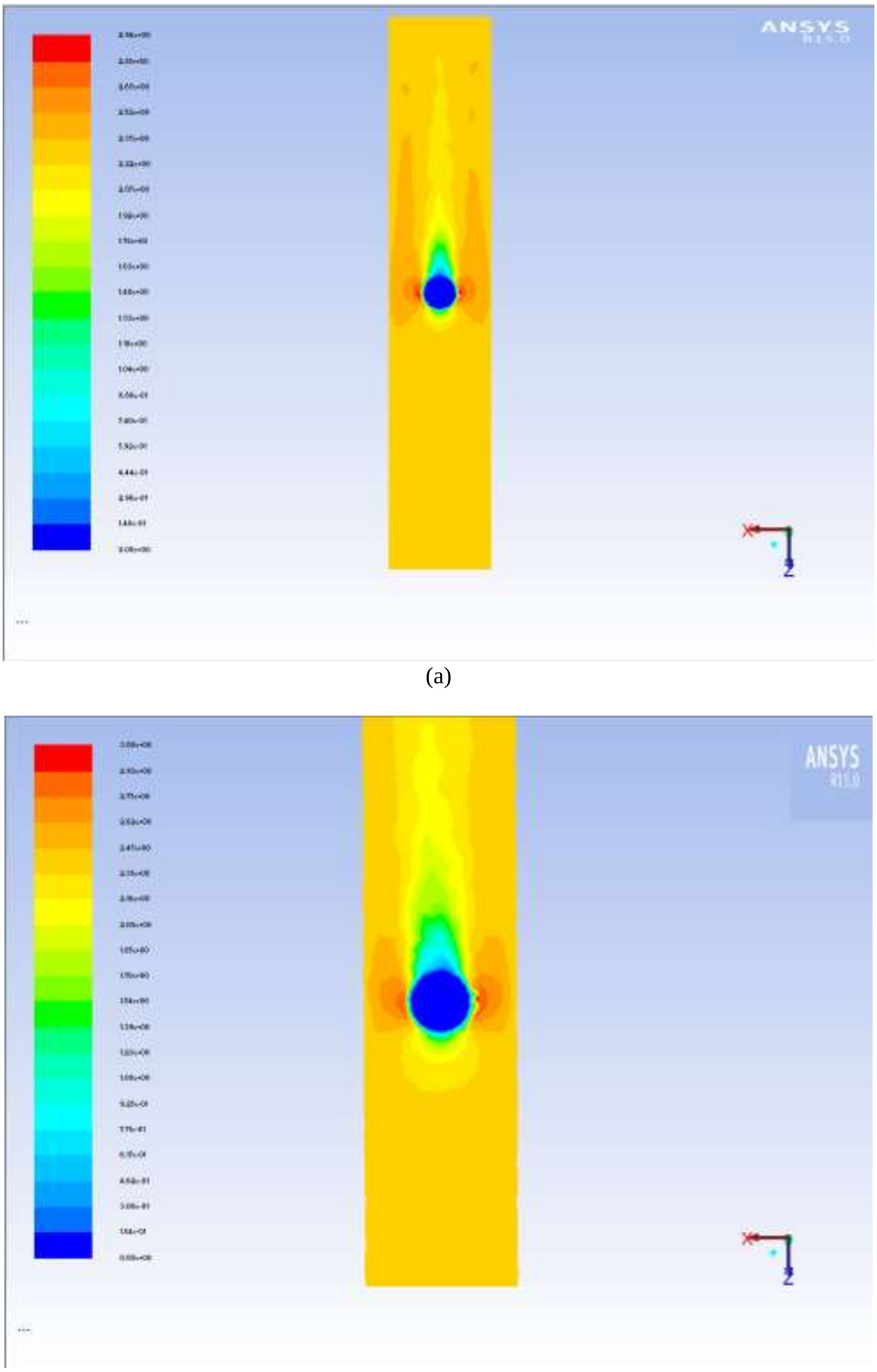


Figure (9): Relationship of (FW-d/D) for falling rectangular in CMC

Behavior of wake region

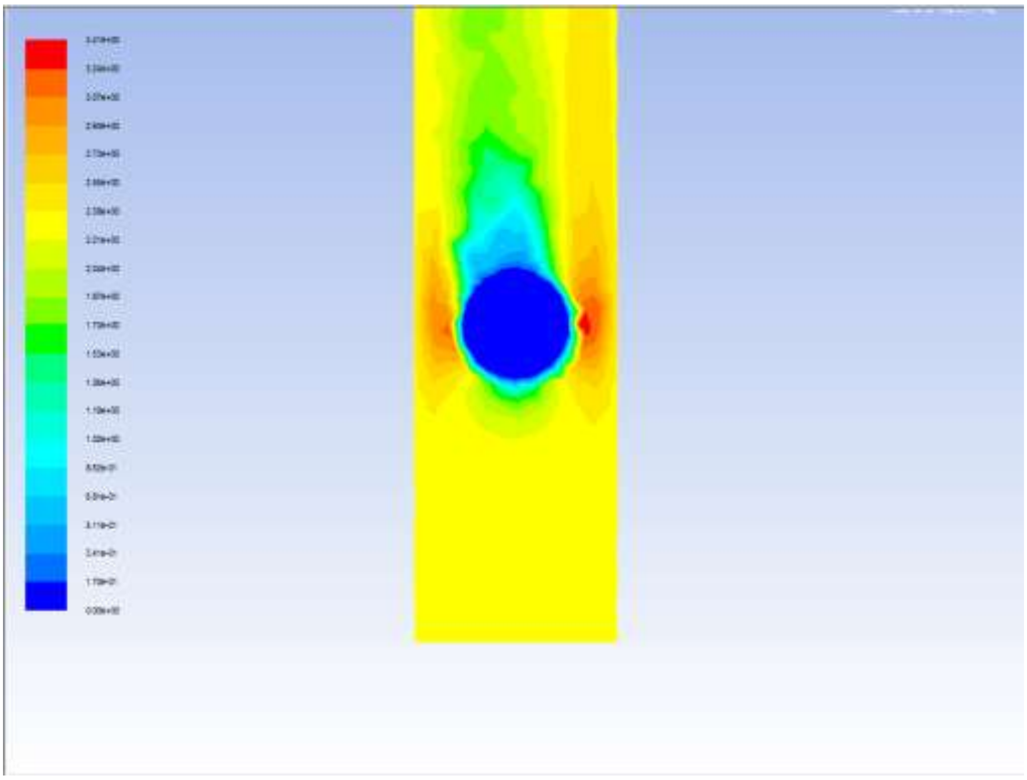
From figures (10)up to (19) show the velocity field contour the simulation appear the boundary layer velocity decrease with increase diameter ratio that the velocity field contour change from orange color to green color Although the use of a single value for the speed in which the value of the terminal velocity in four pipes Calculated from equation (7) .



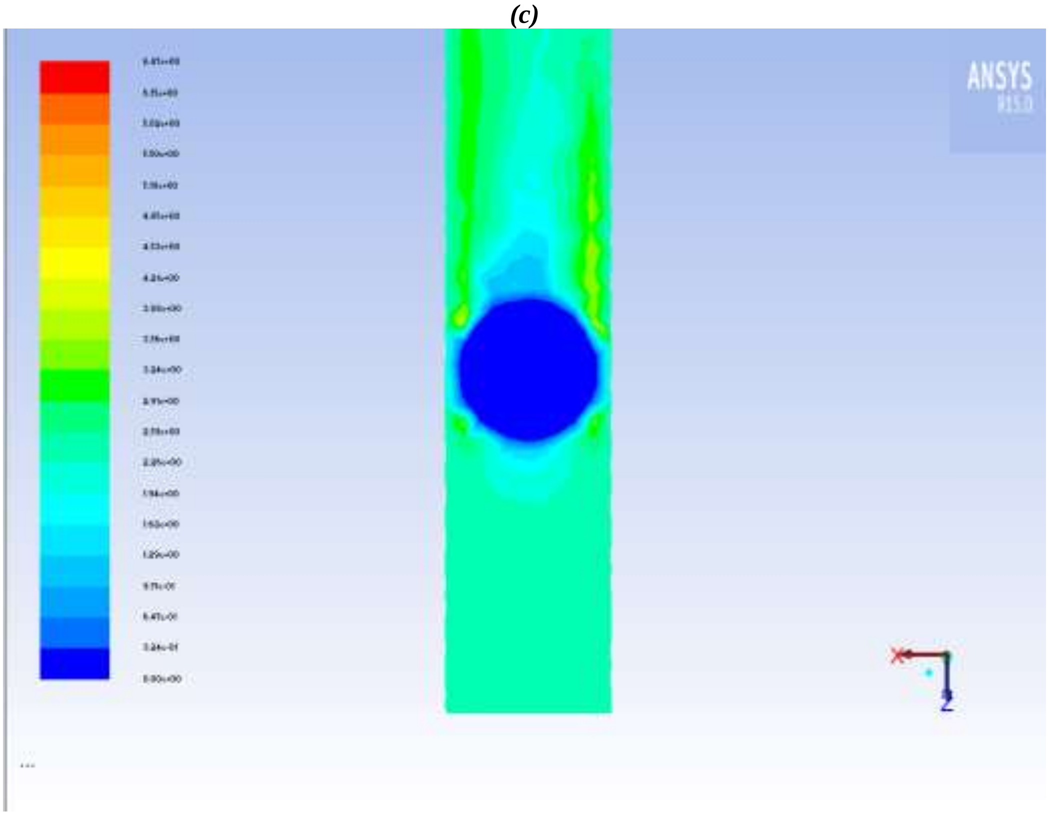
(a)

(b)

Figure (10): sphere (96 g) falling in (a) tube 10cm (b) tube8 cm in CMC

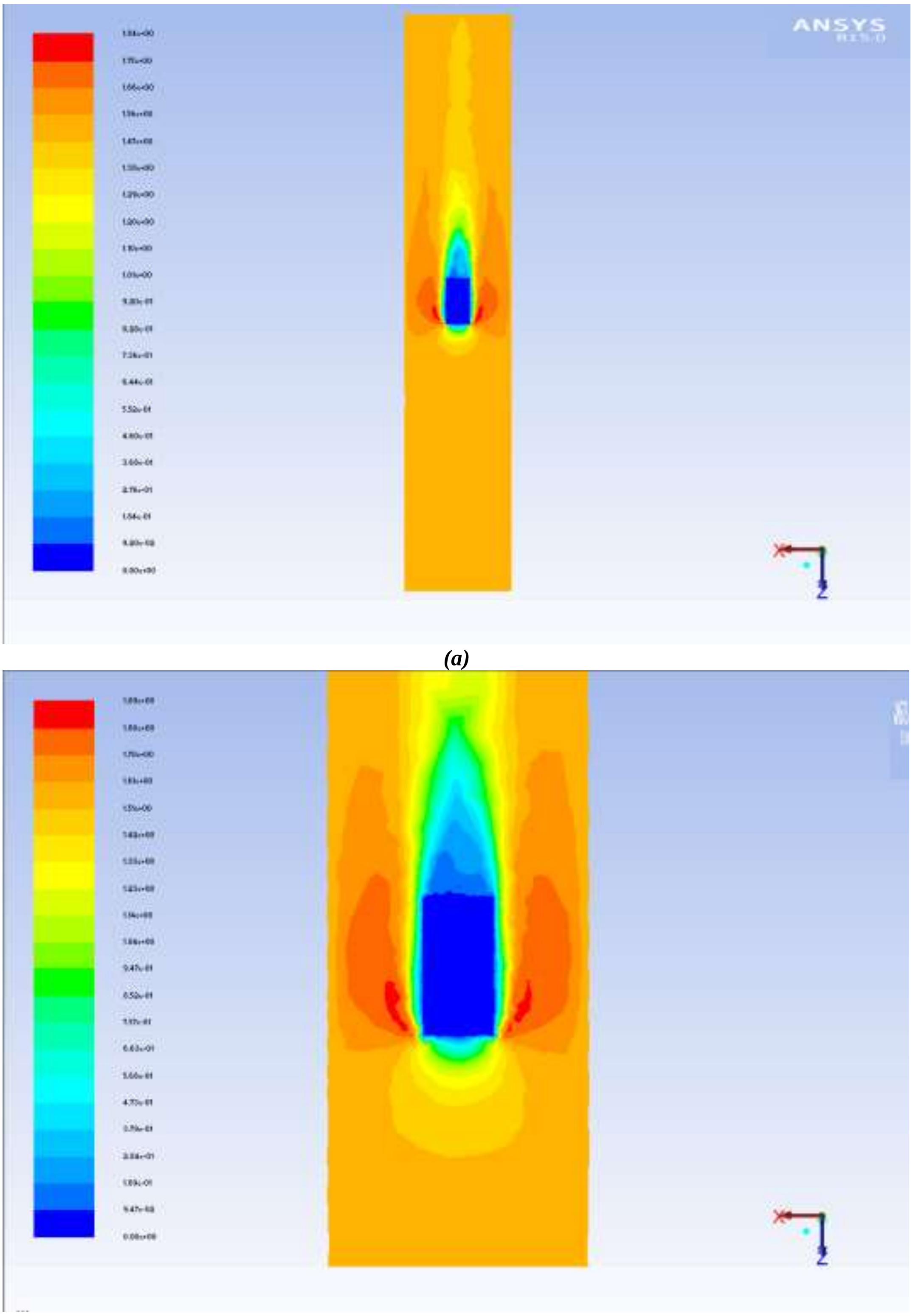


(c)



(d)

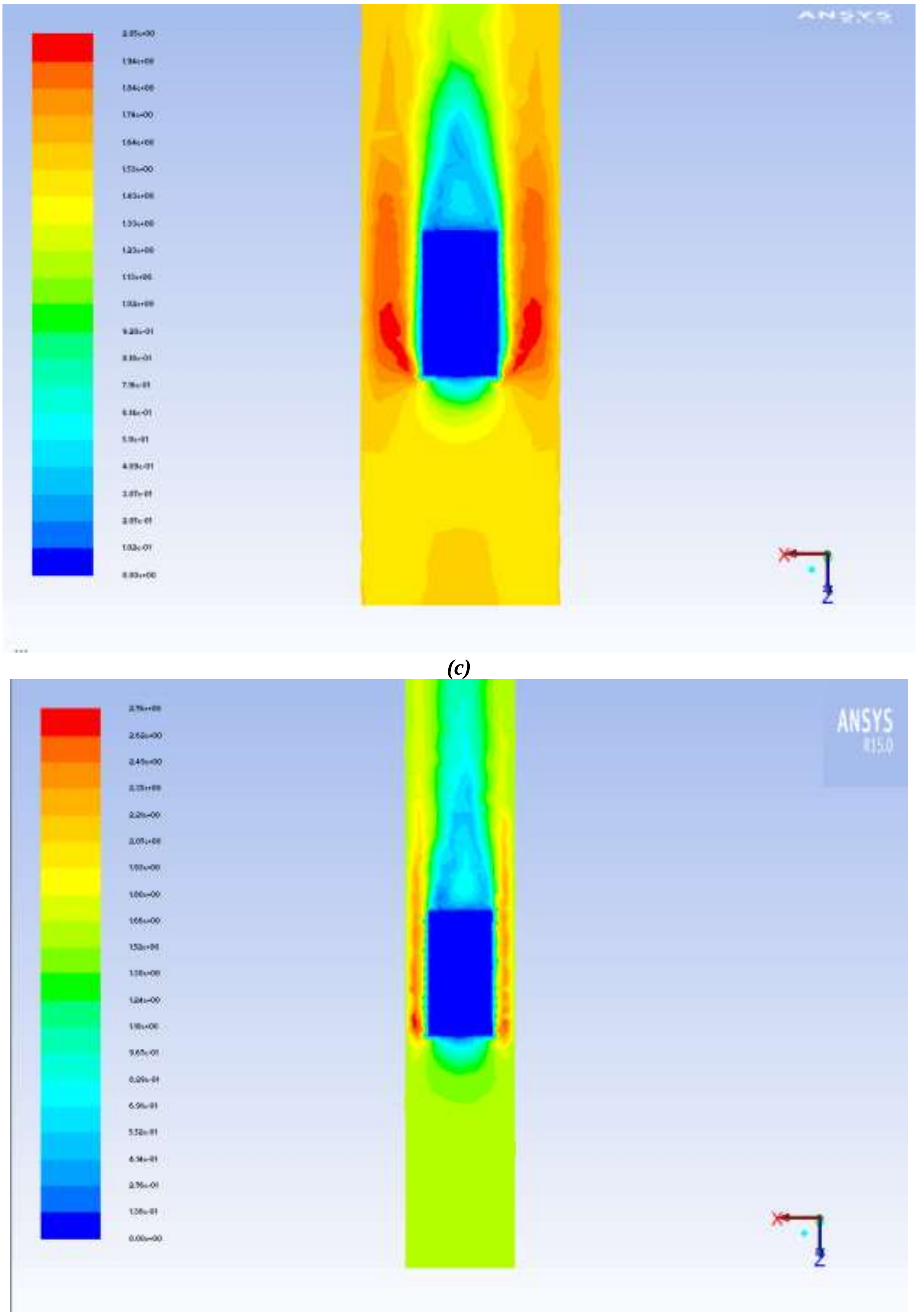
Figure (11): sphere (96 g) falling in (c) tube 6cm (d) tube4 cm in CMC



(a)

(b)

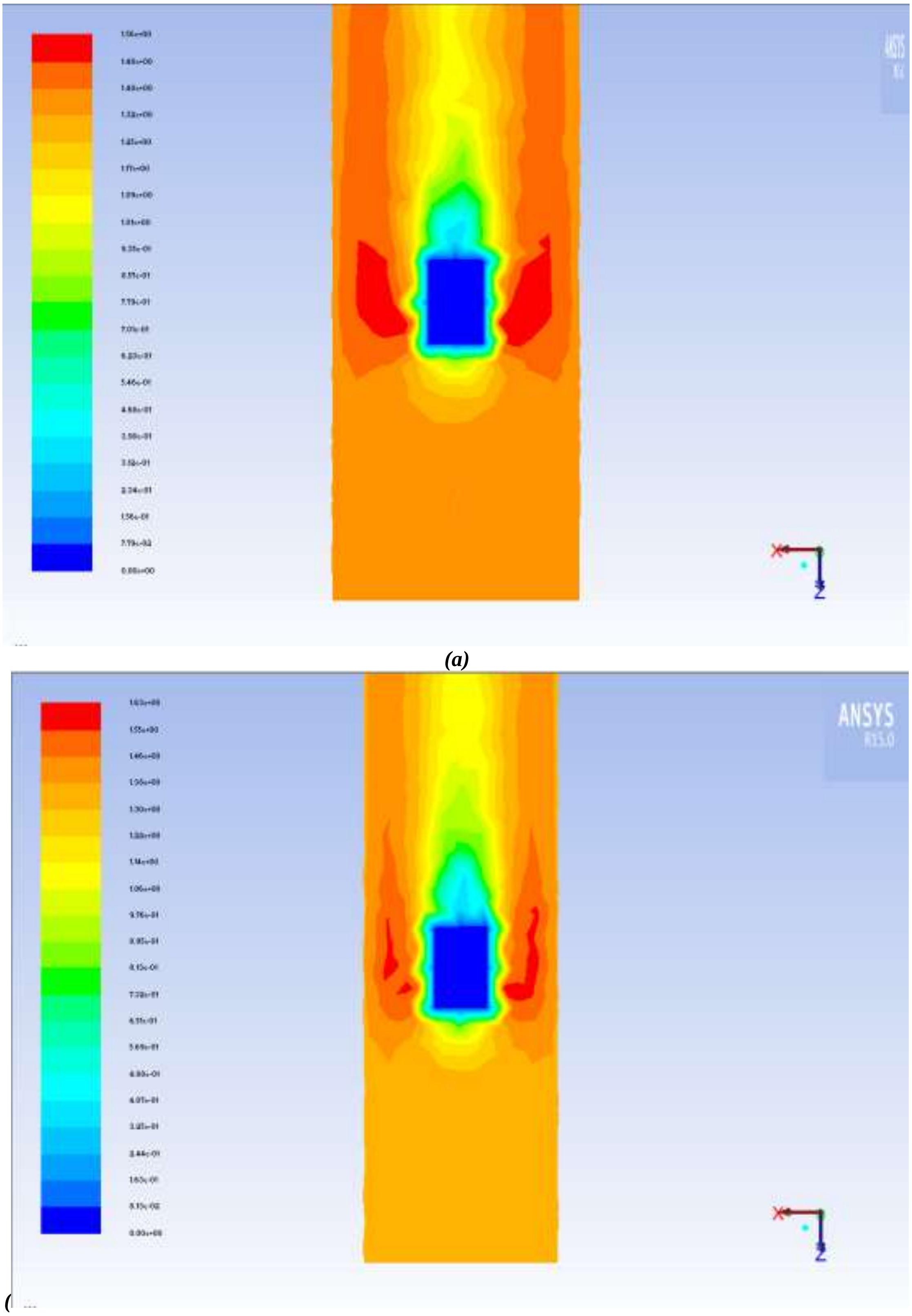
Figure (12): cylinder (96 g) falling in (a) tube 10cm (b) tube8 cm in CMC



(c)

(d)

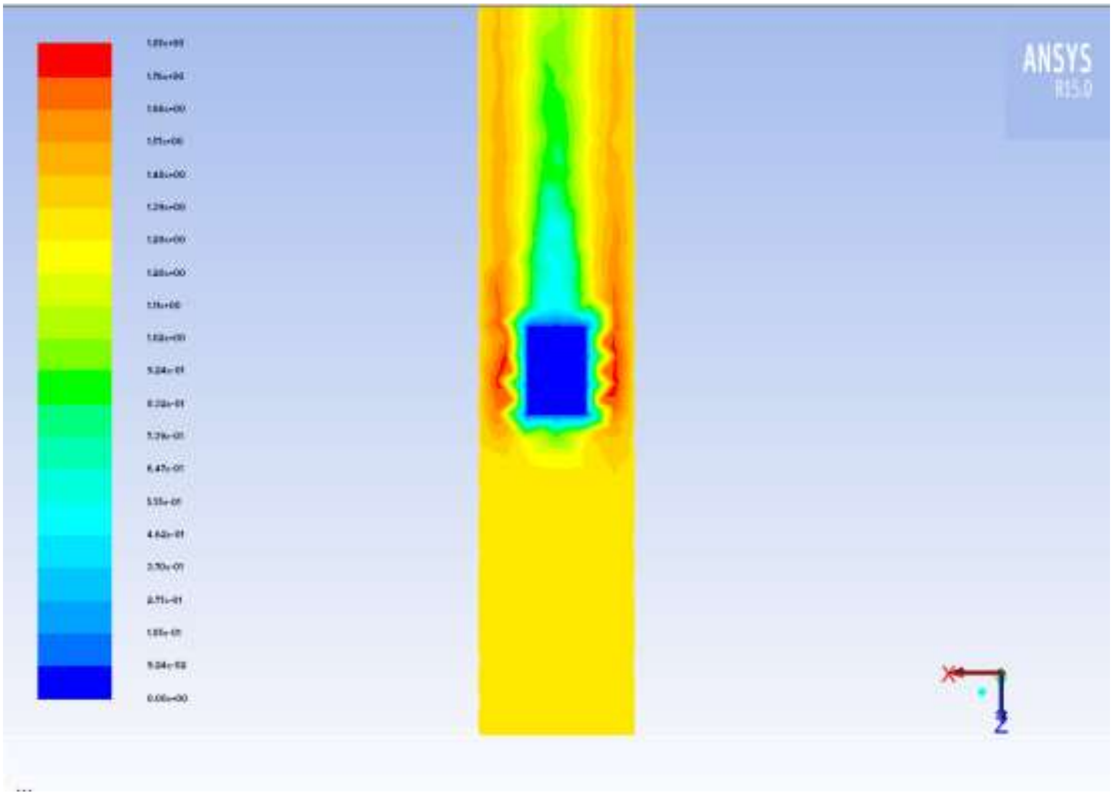
Figure (13): cylinder (96 g) falling in (c) tube 6cm (d) tube4 cm in CMC



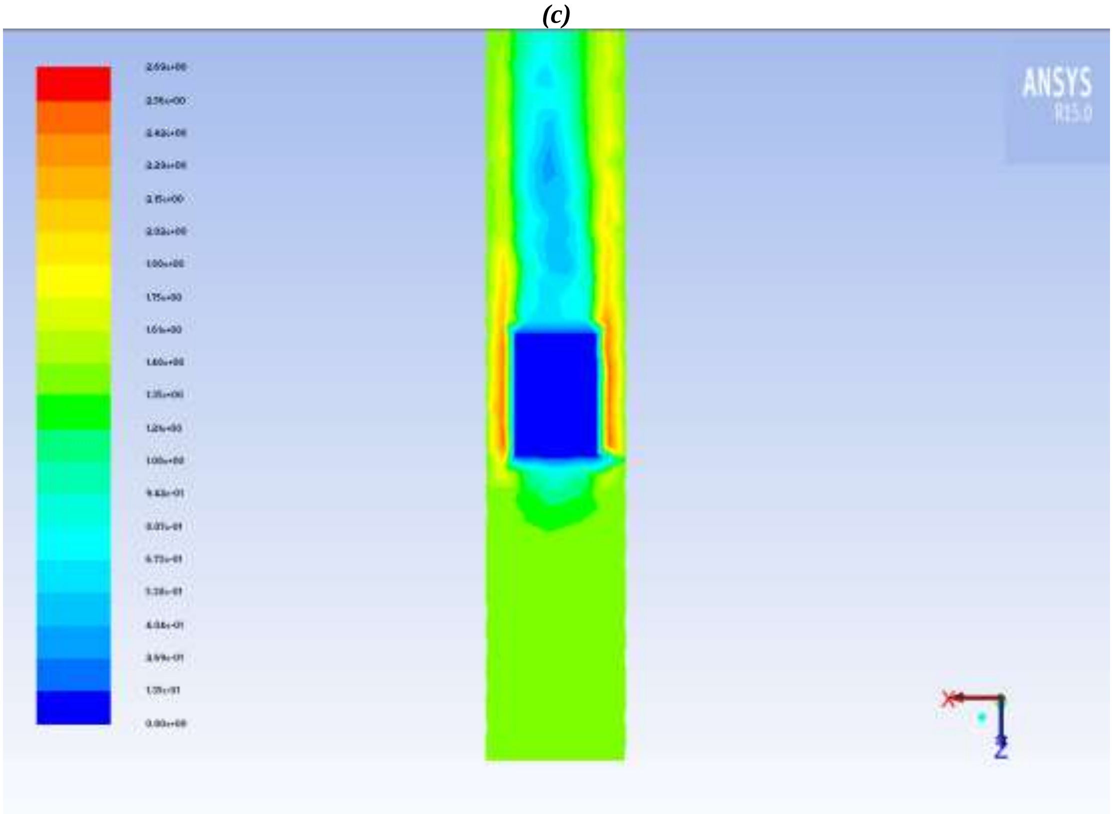
(a)

(b)

Figure (14): rectangular (96 g) falling in (a) tube 10cm (b) tube8cm in CMC

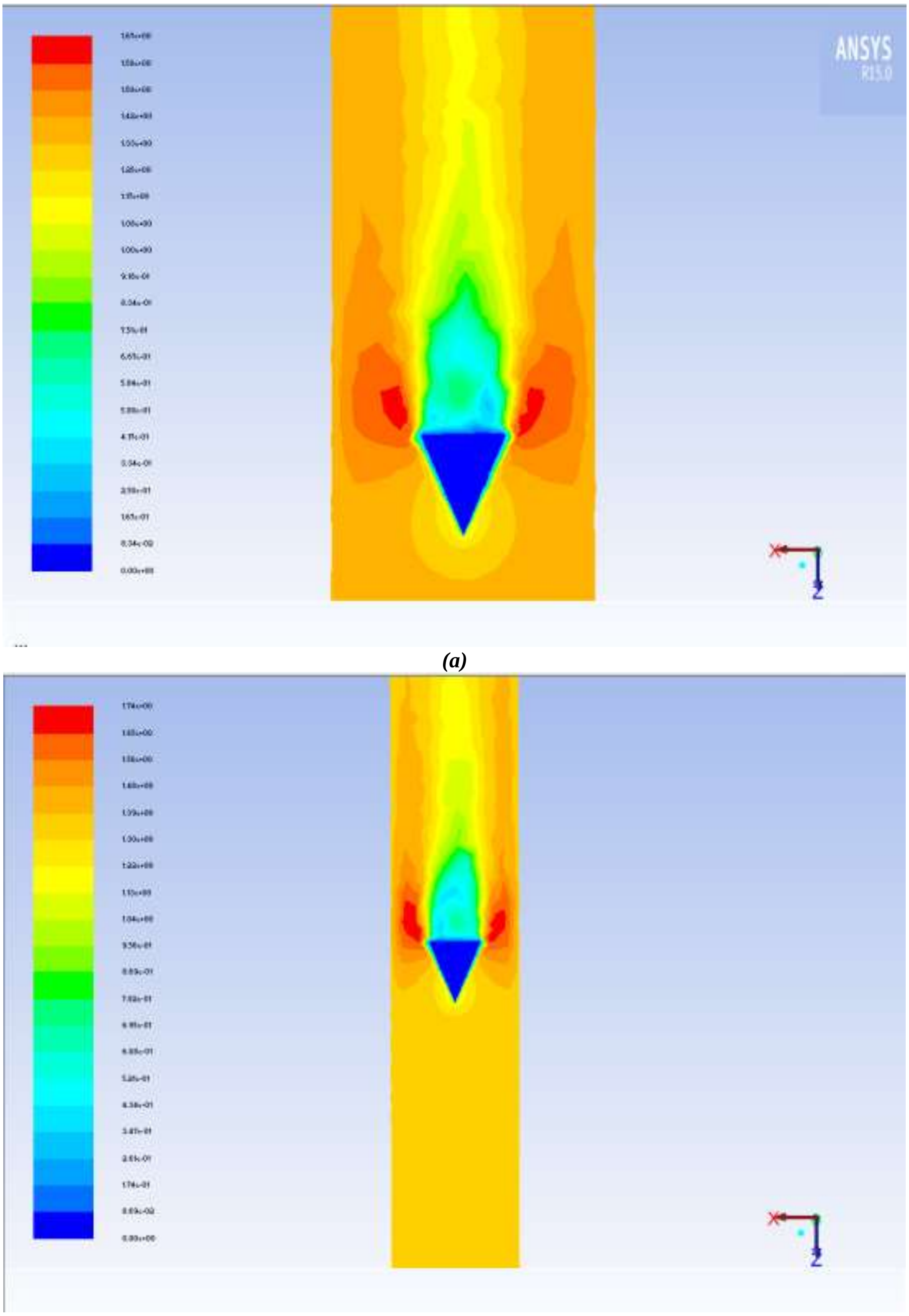


(c)



(d)

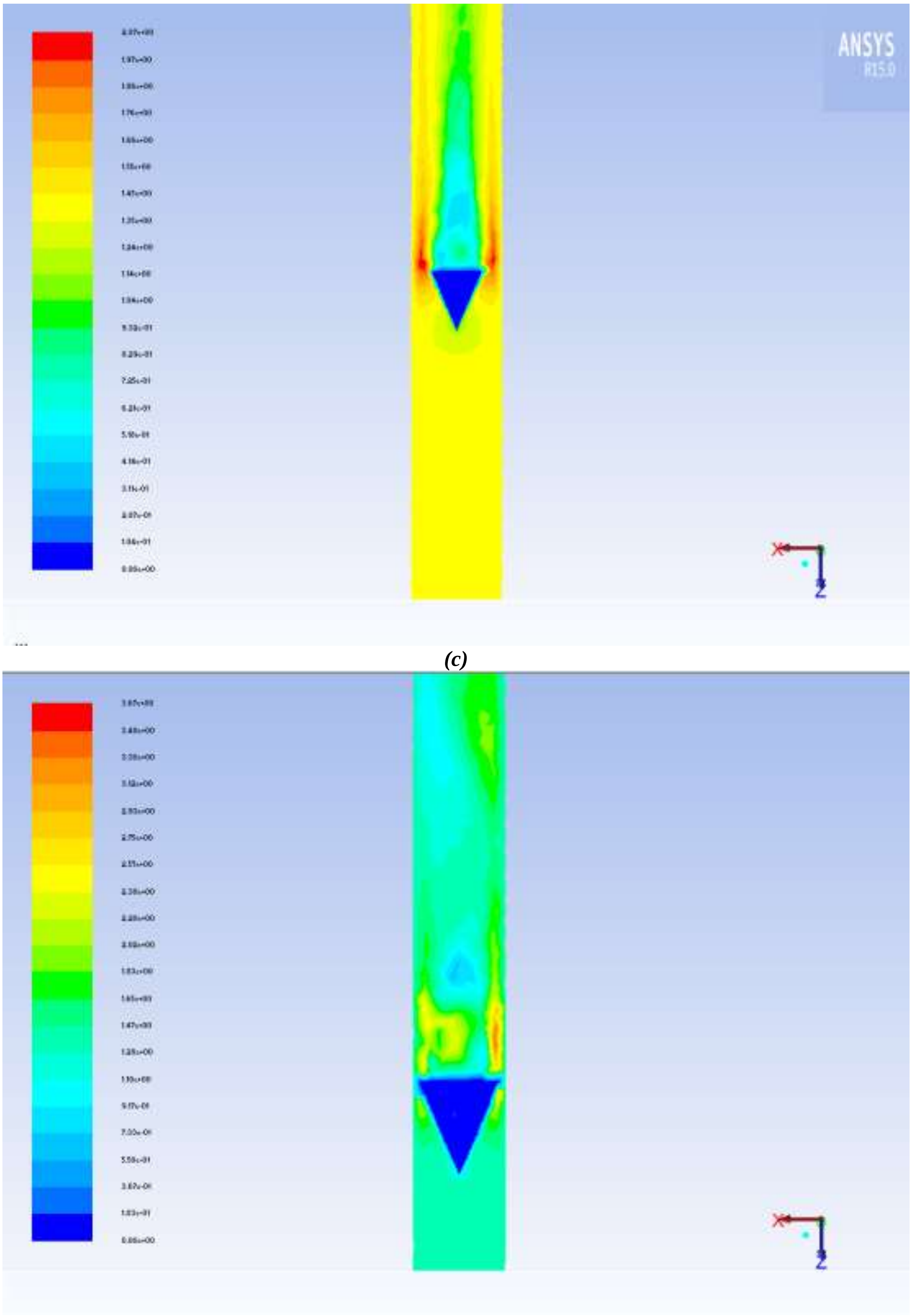
Figure (15): rectangular (96 g) falling in (c) tube 6cm (d) tube4 cm in CMC



(a)

(b)

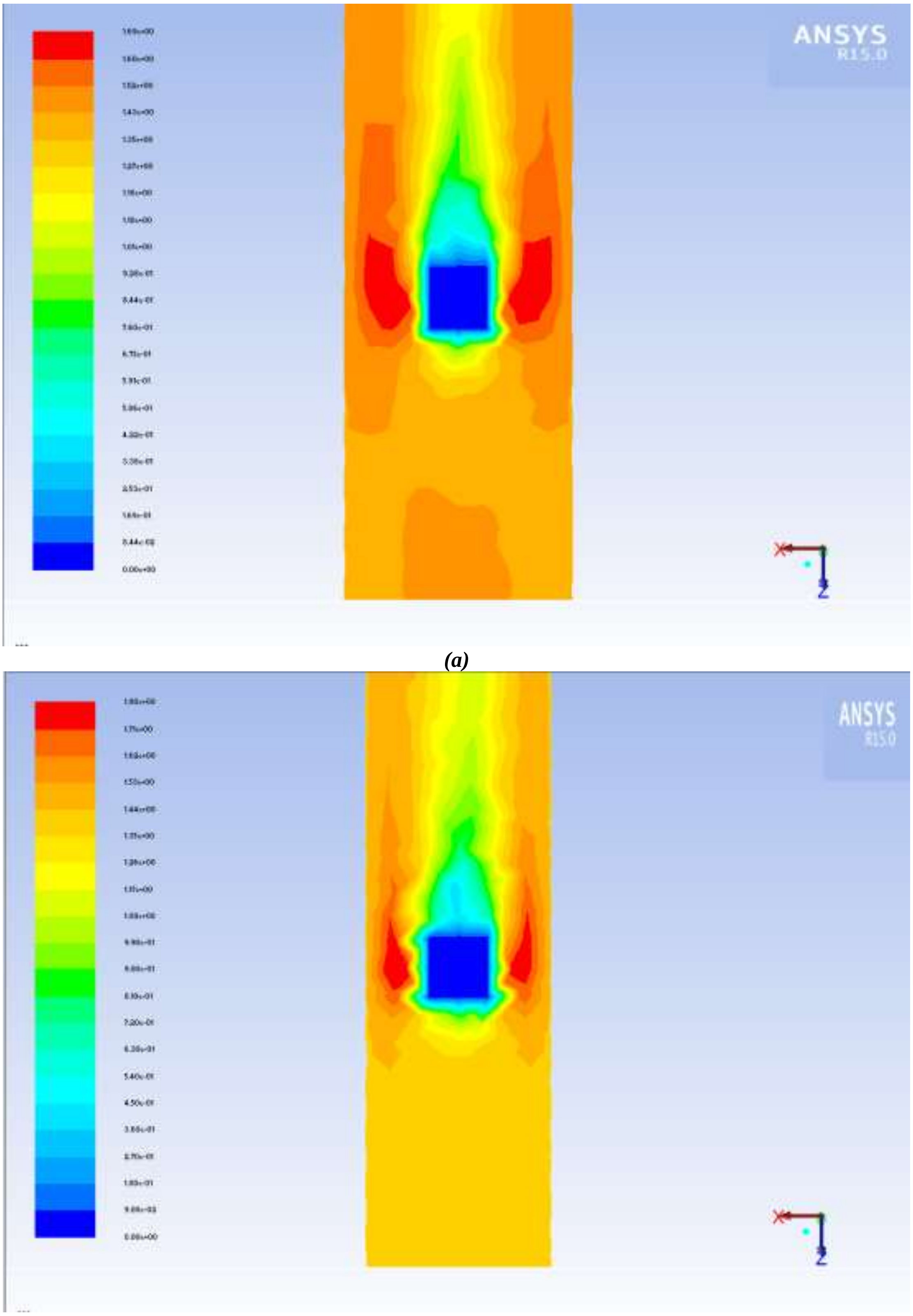
Figure (16): cone (96 g) falling in (a) tube 10cm (b) tube8 cm in CMC



(c)

(d)

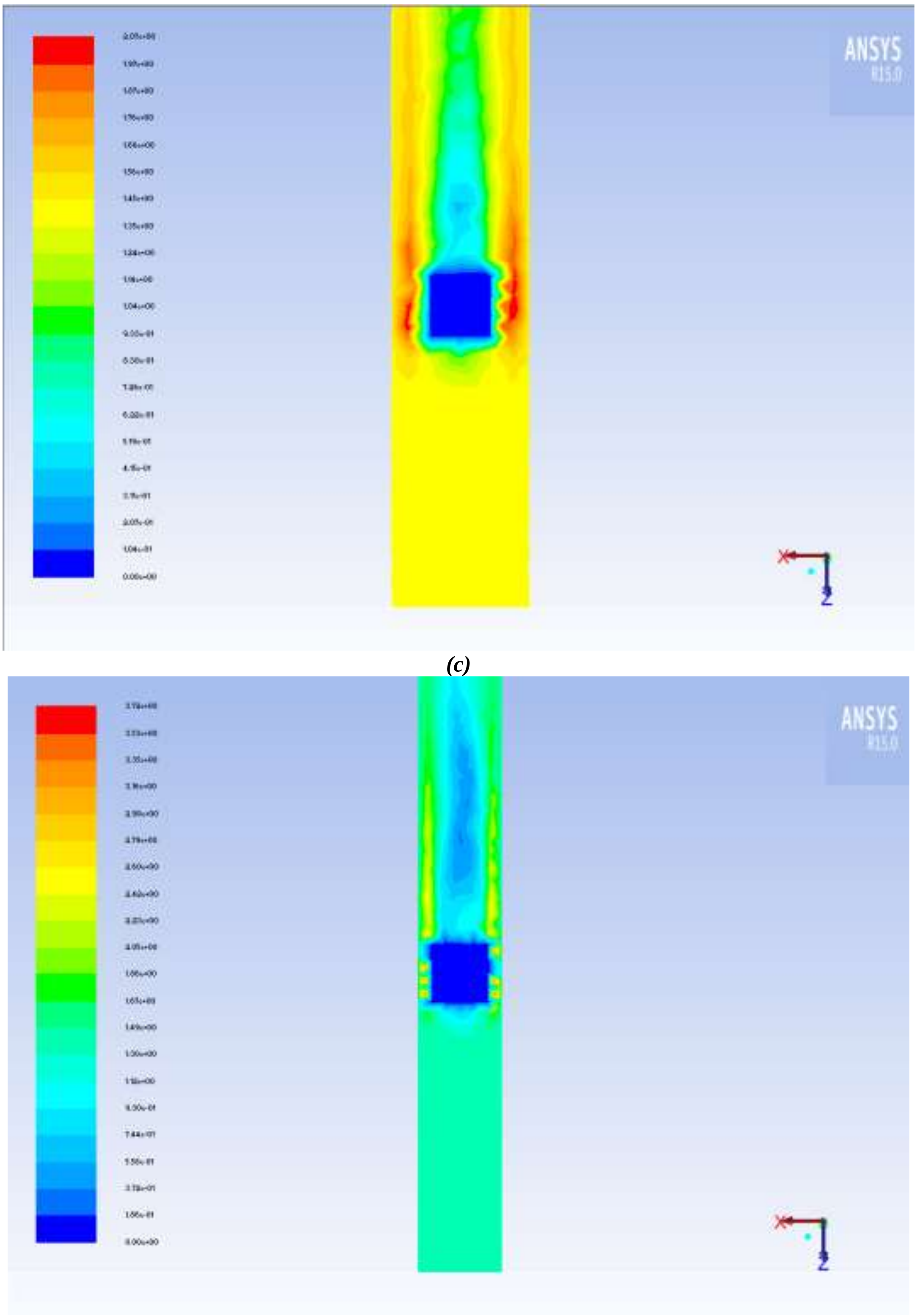
Figure (17): cone (96 g) falling in (c) tube 6cm (d) tube4 cm in CMC



(a)

(b)

Figure (18): cube (96 g) falling in (a) tube 10cm (b) tube8 cm in CMC



(c)

(d)

Figure (19): cube (96 g) falling in (c) tube 6cm (d) tube4 cm in CMC

CONCLUSION

1. The wall correction factor effect increase with increasing diameter ratio.
2. Terminal velocity for particle depend on diameter for particle ,sphericity.
3. The value for drag force increase with increasing diameter ratio.
4. The velocity field contour change from orange color to green color ,this mean the velocity decrease with increasing diameter ratio.

LIST OF SYMBOLS

CD : drag coefficient in bounded medium.
D : diameter for tube (m).
d/D: diameter ratio for particle diameter to tube diameter.
Fw : wall correction factor.
FD: drag force in bounded medium .
FD_∞:drag force in an unbounded medium.
U:velocity for particle(m/s).
Dn: nominal diameter for particle(m).
Ø:sphericity for particle.
m:mass for particle (Kg).
n:flow behavior index.
F_{BE}:Basset history force (N).
C_A: Integrated added mass coefficient.
v:volume for particle (m^3).
Re : Reynolds number .
g : Gravitational acceleration, m/s².
K:consistency index ($pa.s^n$).

GREEK LETTERS

ρ:density of fluid (kg/m^3).
Ϻ:viscosity (pa s).
ρ_p : *Density of particle*,(Kg/m^3).

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